LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **STATISTICS**

THIRD SEMESTER – **APRIL 2023**

UST 3502 – MATRIX AND LINEAR ALGEBRA

Date: 04-05-2023 Dept. No. Max. : 100 Marks

Time: 01:00 PM - 04:00 PM

SECTION A			
Answer ALL the Questions			
1.	**Define the following** **(5 x 1 = 5)**		
a)	Singular and Non-Singular matrices	K1	CO1
b)	Inverse of a matrix	K1	CO1
c)	Basis of a vector space	K1	CO1
d)	Eigen roots	K1	CO1
e)	Index and Signature	K1	CO1
2.	**Fill in the blanks** **(5 x 1 = 5)**		
a)	A matrix **A** such that $\mathbf{A^2 = I}$ is called____________	K1	CO1
b)	In a determinant the sum of the products of the elements of any row (column) with the cofactors for the corresponding elements of any other row (column) is ___________	K1	CO1
c)	If F is any field, then F is a vector space over __________	K1	CO1
d)	The characteristic roots of a skew-Hermitian matrix are _____________	K1	CO1
e)	A real symmetric matrix A is said to be positive definite if the corresponding form X^TAX is____________	K1	CO1
3.	**True or False** **(5 x 1 = 5)**		
a)	The multiplication of matrices is not always commutative.	K2	CO1
b)	If all the elements of a row (or a column) of a determinant are zero, the value of the determinant is zero.	K2	CO1
c)	The set $W = \{(a,0,b): a,b \in R\}$ is not a subspace of $R^3(R)$.	K2	CO1
d)	Every square matrix satisfies its characteristic equation.	K2	CO1
e)	A real symmetric matrix is positive definite if and only if all its eigen values are positive.	K2	CO1
4.	**Match the following** **(5 x 1 = 5)**		
a)	$A^m = 0$ — Linearly Independent	K2	CO1
b)	The method of solving n equations in n unknowns — Unit modulus	K2	CO1
c)	The vectors in a basis are — Index of the nilpotent matrix A	K2	CO1
d)	The characteristic roots of an orthogonal matrix are — $3a^2+7ab+4b^2$	K2	CO1
e)	Real quadratic form — Cramer's rule	K2	CO1

SECTION B			
Answer any TWO of the following questions			**(2 x 10 = 20)**
5.	Solve the following system of linear equations by Cramer's rule: x+y+z=7, x+2y+3z=16, x+3y+4z=22.	K3	CO2
6.	Reduce the matrix $A=\begin{bmatrix}2 & -2 & 0 & 6\\ 4 & 2 & 0 & 2\\ 1 & -1 & 0 & 3\\ 1 & -2 & 1 & 2\end{bmatrix}$ to normal form and find its rank.	K3	CO2
7.	If S, T are two subsets of a vector space V, then prove that (i) $S\subseteq T\Rightarrow L(S)\subseteq L(T)$ (ii) $L(SUT)=L(S)+L(T)$ (iii) $L[L(S)]=L(S)$.	K3	CO2
8.	Verify that the matrix $A=\begin{bmatrix}1 & 2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}$ satisfies its characteristic equation and compute A^{-1}.	K3	CO2
SECTION C			
Answer any TWO of the following questions			**(2 x 10 = 20)**
9.	Prove that every square matrix is uniquely expressible as the sum of a symmetric matrix and a skew-symmetric matrix.	K4	CO3
10.	Compute the inverse of the matrix $A=\begin{bmatrix}1 & 2 & 1\\ 3 & 2 & 3\\ 1 & 1 & 2\end{bmatrix}$ by using E-transformations.	K4	CO3
11.	Consider the basis $S=\{\alpha_1,\alpha_2,\alpha_3\}$ of R^3 where α_1=(1,1,1), α_2=(1,1,0), α_3= (1,0,0). Express (2,-3,5) in terms of the basis elements $\alpha_1,\alpha_2,\alpha_3$.	K4	CO3
12.	Determine a non-singular matrix P such that P^TAP is a diagonal matrix, where $A=\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$.	K4	CO3
SECTION D			
Answer any ONE of the following question			**(1 x 20 = 20)**
13.	Solve the system of equations 2x-2y+5z+3w=0, 4x-y+z+w=0, 3x-2y+3z+4w=0, x-3y+7z+6w=0.	K5	CO4
14.	(i) Evaluate $\Delta=\begin{vmatrix}a & b & c & d\\ b & a & d & c\\ c & d & a & b\\ d & c & b & a\end{vmatrix}$ (10+10) (ii) Write the polynomial $f(x)=x^2+4x-3$ over R as a linear combination of the polynomials $f_1(x)=x^2-2x+5$, $f_2(x)=2x^2-3x$ and $f_3(x)=x+3$.	K5	CO4
SECTION E			
Answer any ONE of the following question			**(1 x 20 = 20)**
15.	Reduce the following quadratic form to canonical form and find its rank and signature $x^2+4y^2+9z^2+t^2-12yz+6zx-4xy-2xt-6zt$.	K6	CO5

16.	(i) Determine the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (12+8) (ii) Justify that the matrix $\begin{bmatrix} 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$ is orthogonal.	K6	CO5

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